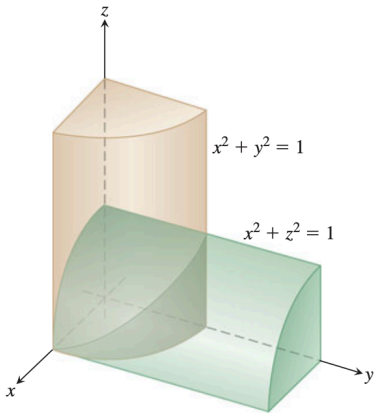
Assignment 4 Solution

§15.5 Q29

Find the volumes of the following region:

The region common to the interiors of the cylinders $x^2 + y^2$ and $x^2 + z^2 = 1$, one, one-eighth of which is shown in the accompanying figure



Solution: Because of symmetry, the volume of the required region is given by

$$8 \times \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2}} dz\, dy\, dx = \frac{16}{3}$$

§15.5 Q42

Evaluate the integral by changing the order of integration in an appropriate way

$$\int_0^1 \int_0^1 \int_{x^2}^1 12xze^{zy^2}\, dy\, dx\, dz$$

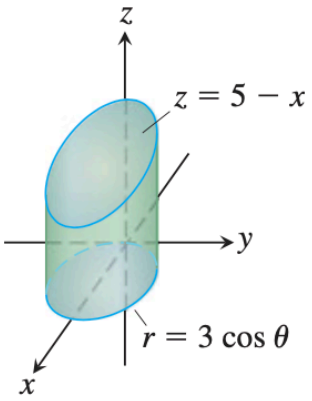
Solution:

$$\int_0^1 \int_0^1 \int_{x^2}^1 12xze^{zy^2}\, dy\, dx\, dz = \int_0^1 \int_0^1 \int_0^{\sqrt{y}} 12xze^{zy^2}\, dx\, dy\, dz = 3e - 6$$

§15.7 Q16

In the following exercise, set up the iterated integral for evaluating $\iiint_D f(r, \theta, z)\, dz\, r\, dr\, d\theta$ over the given region D.

D is the right circular cylinder whose base is the circle $r = 3\cos\theta$ and whose top lies in the plane $z = 5 - x$.



Solution:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{3\cos\theta} \int_{0}^{5-r\cos\theta} dz\ r\ dr\ d\theta$$